5. SUBBOTIN A. I. and CHENTSOV A. G., *Optimization of Guarantee in Control Problems*. Nauka, Moscow, 1981.
6. COURANT R., *Partial Differential Equations*. Mir, Moscow, 1964.
7. MELIKYAN A. A., Necessary optimality conditions on a discontinuity surface of one type in a differential game. *Izv. Akad. Nauk SSSR, Tekh. Kibern.* **4**, 10–18, 1981.

Translated by Z.L.

J. Appl. Maths Mechs Vol. 55, No. 5, pp. 618–625, 1991
Printed in Great Britain.
0021-8928/91 $15.00+.00

TWO PROBLEMS OF ENCOUNTER UNDER CONDITIONS OF UNCERTAINTY†

A. V. PLOTNIKOV and L. I. PLOTNIKOVA

Odessa

(*Received* 3 *July* 1990)

Two problems of the encounter of several controlled objects described by nonlinear differential inclusions, with controls in the right-hand side are considered. Necessary conditions of optimality are obtained in the form of a maximum principle. Previously such problems have been considered for the one-dimensional case [1] and for the multidimensional linear case.‡

1. LET R^n be the n-dimensional real Euclidean space with the norm $\|x\| = (x_1{}^2 + \ldots + x_n{}^2)^{1/2}$, $x = (x_1, \ldots, x_n) \in R^n$. We denote by $\operatorname{conv}(R)$ the space of all nonempty compact and convex subsets in R^n. The metric $h(A, B)$ between the sets A, B in $\operatorname{conv}(R^n)$ is defined by the formula

$$h(A, B) = \min\{r \geqslant 0 \mid A \subset B + S_r(0),\ B \subset A + S_r(0)\},$$

where $S_r(a)$ is the sphere in R^n with the radius $z > 0$ centred at the point $a \in R^n$.

Denote by $\operatorname{cc}(R^n)$ the space of all nonempty compact subsets of the space $\operatorname{conv}(R^n)$ with the metric

$$\delta(A, B) = \max\{\max_{a\in A} \min_{b\in B} h(a, b), \max_{b\in B} \min_{a\in A} h(a, b)\}$$

† *Prikl. Mat. Mekh.* Vol. 55, No. 5, pp. 752–758, 1991.

‡ RADZHEF M. S., Investigation of one problem of optimal control of M-objects with multivalued trajectories, Odessa State University, Odessa, 1983. Unpublished manuscript, UkrNIINTI 30.01.84, No. 137–Uk84.

Let $L_n^M[t_0, T]$ be the space measurable and integrable (on $[t_0, T]$) multivalued mappings $F:[t_0, T]\to \text{conv}(R^n)$ with the metric

$$p(F, G) = \int_{t_0}^{T} h(F(t), G(t))\, dt$$

Definition 1 [2]. We say that the sequence $\{F_k(\cdot)\}_{k=1}^{\infty}$ from $L_n^M[t_0, T]$ weakly converges to $F(\cdot)\in L_n^M[t_0, T]$ if one of the following equivalent assertions holds:

1. For each bounded and measurable function $p:[t_0, T]\to R^n$ the sequence of sets

$$\left\{\int_{t_0}^{T} p^T(t)\, F_K(t)\, dt\right\}_{K=1}^{\infty}$$

in R^1 converges to

$$\int_{t_0}^{T} p^T(t) F(t) dt$$

2. For each bounded and measurable function $p:[t_0, T]\to R^n$ the sequence of real numbers

$$\left\{\int_{t_0}^{T} C(F_K(t), p(t))\, dt\right\}_{K=1}^{\infty}$$

converges to

$$\int_{t_0}^{T} C(F(t), p(t)) dt$$

3. For each $p\in R^n$ and measurable subset M from $[t_0, T]$, the sequence

$$\left\{\int_{M} C(F_K(t), p)\, dt\right\}_{K=1}^{\infty}$$

converges to

$$\int_{M} C(F(t), p) dt$$

Suppose that the behaviour of the object is described by the following differential equations with multivalued right-hand side

$$x^{\cdot} \in F(t, x, u),\; x\in R^n,\; u\in R^m \qquad F: R^1\times R^n\times R^m\to \text{conv}(R^n), \tag{1.1}$$

where x is the phase vector, u is the control vector and F is a multivalued mapping (MVM).

We assume that the initial state of the object is given

$$x(t_0) = x^{\circ},\; x^{\circ}\in R^n \tag{1.2}$$

Let the MVM

$$U: R^1\to \text{conv}(R^m) \tag{1.3}$$

be given.

Definition 2. The class LU of admissible controls of the object (1.1), (1.2) consists of all measurable selectors of the MVM (1.3).

Assume that the system (1.1)–(1.3) satisfies the following assumptions.

A1. (a) The MVM $F(\cdot, x, u)$ is measurable with respect to t on R^1 for fixed $(x, u) \in R^n \times R^m$; (b) the MVM $F(t, \cdot, u)$ is continuous and satisfies the Lipschitz condition with the constant L with respect to x on R^n for fixed $(t, u) \in R^1 \times R^m$; (c) the MVM $F(t, x, \cdot)$ is continuous with respect to u on R^m for fixed $(t, x) \in R^1 \times R^n$.

A2. A function $k(\cdot) \in L_1{}^1(R^1)$ exists such that $|F(t, x, u)| \leqslant k(t)$ for almost all $(t, x, u) \in R^1 \times R^n \times R^m$.

A3. The MVM $U: R^1 \rightarrow \mathrm{conv}(R^m)$ is measurable with respect to t on R^1.

A4. A function $l(\cdot) \in L_1{}^1(R^1)$ exists such that $|U(t)| \leqslant l(t)$ for almost all $t \in R^1$.

A5. If the sequence $\{u_K(\cdot)\}_{K=1}^{\infty}$ from LU weakly converges to $u_*(\cdot) \in \mathrm{LU}$, then for any absolutely continuous function $x(\cdot)$ the sequence $\{F(\cdot, x(\cdot), u_K(\cdot))\}_{K=1}^{\infty}$ weakly converges to $F(\cdot, x(\cdot), u_*(\cdot))$.

A6. For almost all $t \in R^1$ and any values $\alpha, \beta \geqslant 0$, $\alpha + \beta = 1$ and points

$$x_1,\quad x_2 \in S_{\int_{t_0}^{t} k(s)ds}(0)$$

we have

$$\alpha R(t, x_1) + \beta R(t, x_2) \subset R(t, \alpha x_1 + \beta x_2)$$
$$\left(R(t, x) \equiv \bigcup_{u \in U(t)} F(t, x, u)\right)$$

and for any $u(\cdot) \in \mathrm{LU}$ we have

$$\alpha F(t, x_1, u(t)) + \beta F(t, x_2, u(t)) \in F(t, \alpha x_1 + \beta x_2, u(t)).$$

A7. The support function $C(F(t, x, u), \psi)$ of the set $F(t, x, u)$ is continuously differentiable with respect to x for almost all $(t, u, \psi) \in R^1 \times R^m \times R^n$.

A8. There exists a function $m(\cdot) \in L_1{}^1(R^1)$ such that for any two vectors $\psi_1, \psi_2 \in R^n$, we have the inequality

$$\left| \frac{\partial C(F(t, x, u), \psi_1)}{\partial x} - \frac{\partial C(F(t, x, u), \psi_2)}{\partial x} \right| \leqslant m(t)\,|\psi_1 - \psi_2|$$

Definition 3. The multivalued trajectory of system (1.1), (1.2) corresponding to the admissible control $u(\cdot) \in \mathrm{LU}$ is the MVM $X(\cdot, u)$ whose value at each instant of time $t \geqslant t_0$ is equal to the section of the solution set of the differential inclusion (1.1) and (1.2) corresponding to the control $u(\cdot) \in \mathrm{LU}$.

Definition 4. The reachability set $Y(T)$ of system (1.1)–(1.3) is the set of all subsets in the space $\mathrm{conv}(R^n)$ which can be reached during the time $[t_0, T]$ from the initial state $x^0 \in R^n$ by the solutions of the differential inclusion (1.1) for all possible admissible controls from the family LU, i.e. $Y(T) = \{X(T, u), u(\cdot) \in \mathrm{LU}\}$.

Theorem 1. Assume that conditionsA1–A6 are satisfied. Then the reachability set $Y(T)$ of system (1.1)–(1.3) is compact, i.e. $Y(T) \in \mathrm{cc}(R^n)$.†

† For a proof see PLOTNIKOV A. V., Compactness of the reachability set of a differential inclusion containing a control, Odessa State University, Odessa, 1988. Unpublished manuscript, UkrNIINTI 11.05.88, No. 1145–Uk88.

2. Consider $N(N \geqslant 2)$ controlled objects whose behaviour is described by differential inclusions with control

$$x_i^{\cdot} \in F_i\ (t,\ x_i,\ u_i);\ \ x_i \in R^n,\ \ u_i \in R^{m_i} \qquad F_i\colon R^1 \times R^n \times R^{m_i} \to \operatorname{conv}\ (R^n) \tag{2.1}$$

where x_i is the phase vector, u_i is the control vector and F_i is a MVM. Here and henceforth, $i = 1, 2, \ldots, N$.

The initial position of each object is known

$$x_i\ (t_0) = x_i^{\circ} \tag{2.2}$$

Let the MVM

$$U_i\colon R^1 \to \operatorname{conv}\ (R^{m_i}) \tag{2.3}$$

be given. Denote by LU_i the set of admissible controls of object i. Assume that the MVMs $F_i(\cdot, \cdot, \cdot)$ and $U_i(\cdot)$ satisfy conditions A1–A8.

Consider the following problems of optimal guidance of all N objects to the same point (or the same set) in the phase space R^n.

Problem 1. Find the times $T_i > t_0$ and the controls $u_i(\cdot) \in \mathrm{LU}_i$ such that for the corresponding multivalues trajectories $X_i(\cdot, u_i)$ of system (2.1)–(2.3): first, the intersection of the sets $X_i(T_i, u_i)$ is nonempty, i.e.

$$X_1\ (T_1,\ u_1) \cap \ldots \cap X_N\ (T_N,\ u_N) \neq \varnothing \tag{2.4}$$

and, second, the criterion

$$J = T,\ \ T = \max\ \{T_i\} \tag{2.5}$$

attains its minimum.

Definition 5. The collection of admissible controls $u_* = (u_{1_*}, \ldots, u_{n_*})$ is called optimal if the corresponding multivalued trajectories $X_i(\cdot, u_{i_*})$ of the system (2.1)–(2.3) satisfy the following relationships:

1. $X_1\ (T_1,\ u_{1*}) \cap \ldots \cap X_N\ (T_N,\ u_{N*}) \neq \varnothing;$
2. For any $J \in \{1, \ldots, N\}$ for $\tau < T_J$ we have

$$X_1\ (T_1,\ u_{1*}) \cap \ldots \cap X_j\ (\tau_J,\ u_J) \cap \ldots \cap X_N\ (T_N,\ u_{N*}) \neq \varnothing, \qquad \forall u_j\ (\cdot) \in \mathrm{LU}_J$$

In this case, the multivalued trajectories $X_i(\cdot, u_{i_*})$ are called optimal.

Definition 6. We say that the pairs $(u_{i_*}(\cdot), X_i(\cdot, u_{i_*}))$ satisfy the maximum principle in the respective intervals $[t_0, T]$ if there exist vector functions $\psi_{i_*}(\cdot)$ which are nontrivial solutions of the corresponding differential equations

$$\psi_i^{\cdot} = -\frac{\partial C\ (F_i\ (t,\ x_{i*}\ (t),\ u_{i*}\ (t)),\ \psi_i)}{\partial x} \tag{2.6}$$

and the following conditions are satisfied:

1. The maximum condition

$$C\,(F_i\,(t,\,x_{i*}\,(t),\,u_{i*}\,(t)),\,\psi_{i*}\,(t)) = \max_{u_i \in U_i(t)} C\,(F_i\,(t, x_{i*}\,(t), u_i), \psi_{i*}\,(t)), \tag{2.7}$$

where $x_{i_*}(\cdot)$ is the solution of the equation

$$(x_{i*}^{\cdot}\,(t), \psi_{i*}\,(t)) = \frac{\partial C\,(X_i\,(t, u_{i*}), \psi_{i*}\,(t))}{\partial t} = C\,(F_i\,(t, x_{i*}\,(t),\,u_{i*}\,(t)), \psi_{i*}\,(t))$$

for almost all $t \in [t_0, T_i]$.

2. The transversality condition

$$C\,(X_i\,(T_i, u_{i*}), \psi_{i*}\,(T_i)) = -C\,(\bigcap_{i=1}^{N} X_i\,(T_i, u_{i*}), -\psi_{i*}\,(T_i))$$

Thorem 2. Assume that in Problem 1 the collection of admissible controls $u_*(\cdot) = \{u_{1_*}(\cdot), \ldots, u_{N_*}(\cdot)\}$ is optimal and $X_i(\cdot, u_{i_*})$ are the corresponding multivalued trajectories of system (2.1)–(2.3). Then the pairs $(u_{i_*}(\cdot), X_i(\cdot, u_{i_*}))$ satisfy the maximum principle in the intervals $[t_0, T_i]$.

Proof. Let $u_{i_*}(\cdot) \in \mathrm{LU}_i$ be the optimal control of object i and $X_i(\cdot, u_{i_*})$ the corresponding multivalued trajectory. Then

$$X_i\,(T_i,\,u_{i*}) \in Y_i\,(T_i),\;\; X_i\,(T_i,\,u_{i*}) \cap S_k \neq \varnothing \tag{2.8}$$
$$S_K = \bigcap_{i=1}^{N} X_i\,(T_i, u_{i*})$$

where $Y_i(T_i)$ is the reachability set of object i in time T_i.

Therefore, from (2.8) and Assumption A6 we have

$$\max_{X_i \in Y_i(T_i)} C\,(X_i, \psi_i) + C\,(S_K, -\psi_i) \geqslant 0, \quad \forall \psi_i \in R^n \tag{2.9}$$

If we put

$$\omega_i = \max_{X_i \in Y_i(T_i)} \min_{\psi_i \in S_1(0)} [C\,(X_i, \psi_i) + C\,(S_K, -\psi_i)], \tag{2.10}$$

then $\omega_i \geqslant 0$. Indeed, if $\omega_i < 0$, then there exists a set $X_i \in Y_i(T_i)$ such that for all $\psi_i \in S_l(0)$ we have the inequality

$$C\,(X_i,\,\psi_i) + C\,(S_K,\,-\psi_i) \geqslant 0$$

[this contradicts inequality (2.9)].

We will show that $\omega_i = 0$ and equality to zero is achieved for some $X_i = X_i(T_i, u_{i_*})$ and some $\psi_i = \psi_{i_*}^{\circ} \in S_1(0)$.

Indeed, from the relationship $X_i(T_i, u_{i_*}) \cap S_K \neq \varnothing$ it follows that

$$\omega_i = C\,(X_i\,(T_i,\,u_{i*}),\,\psi_i) + C\,(S_K,\,\,-\psi_i) \geqslant 0,\;\, \forall \psi_i \in S_1\,(0)$$

The function $\omega_i(T_i, \psi_i)$ is continuous in T_i and ψ_i by the continuity of the support functions and the set $Y_i(T_i)$.

If we assume that

$$\omega_i (T_i, \psi_i) > 0, \ \forall \psi_i \in S_1 (0)$$

and therefore $\omega_i \neq 0$, we obtain

$$\omega_i^\circ (T_i) = \min_{\psi_i \in S_1(0)} \omega_i (T_i, \psi_i) \geqslant \alpha_i > 0$$

and the function $\omega_i^\circ(T_i)$ is continuous. Thus, there exists $\tau_i < T_i$ such that $\omega_i^\circ(\tau_i) \geqslant 0$. This means that

$$C (X_i (\tau_i, u_{i*}), \psi_i) + C (S_K, -\psi_i) \geqslant 0, \ \forall \psi_i \in S_1 (0),$$

i.e. $X_i(\tau_i, u_{i_*}) \cap S_K \neq \varnothing$, which contradicts the optimality of the trajectory $X_i(\cdot, u_{i_*})$.

If we assume that $\omega_i > 0$ and the maximum in (2.10) is attained for $X_i \neq X_i(T_i, u_{i_*})$, then, as above, we obtain a contradiction.

Thus, a vector $\psi_i = \psi_i^\circ$ exists such that

$$C (X_i (T_i, u_{i*}), \psi_i^\circ) = \max_{X_i \in Y_i(T_i)} C (X_i, \psi_i^\circ) \tag{2.11}$$

$$C (X_i (T_i, u_{i*}), \psi_i^\circ) = -C (S_K, -\psi_i^\circ) \tag{2.12}$$

In other words, a vector $x_i^\circ \in \partial X_i(T_i, u_{i_*})$ exists which satisfies the equalities

$$(x_i^\circ, \psi_i^\circ) = \max_{X_i \in Y_i(T_i)} C (X_i, \psi_i^\circ) = C (X_i (T_i, u_{i*}), \psi_i^\circ), \tag{2.13}$$

$$(x_i^\circ, \psi_i^\circ) = -C (S_K, -\psi_i^\circ) \tag{2.14}$$

Then a function $x_{i_*}(\cdot)$ exists such that

$$x_{i*} (t) \in \partial X_i (t, u_{i*}), \quad t \in [t_0, T_i] \text{ и } x_{i*} (T_i) = x_i^\circ \tag{2.15}$$

and hence a function $\psi_{i_*}(\cdot) \in S_1(0)$ exists such that $(x_{i_*}(t), \psi_{i_*}(t)) = C(X_i(t, u_{i_*}), \psi_{i_*}(t))$ for almost all $t \in [t_0, T_i]$ and $\psi_{i_*}(T_i) = \psi_i^\circ$.

Clearly $x_{i_*}(\cdot)$ is the boundary solution of the inclusion $\dot{x}_i \in R_i(t, x_i)$ and is therefore an optimal solution of this inclusion. Then, using the well-known result from [3], we can write for almost all $t \in [t_0, T_i]$

$$(\dot{x}_{i*} (t), \psi_{i*} (t)) = C (R_i (t, x_{i*} (t)), \psi_{i*} (t)) \tag{2.16}$$

From (2.15) and (2.16) it follows that for almost all $t \in [t_0, T_i]$

$$C (F_i (t, x_{i*} (t), u_{i*} (t)), \ \psi_{i*} (t)) = C (R_i (t, x_{i*} (t)), \psi_{i*} (t))$$

Hence for almost all $t \in [t_0, T_i]$

$$C (F_i (t, x_{i*} (t), u_{i*} (t)), \psi_{i*} (t)) = \max_{u_i \in U_i(t)} C (F_i (t, x_{i*} (t), u_i), \psi_{i*} (t))$$

Similarly from [3] we obtain that the function $\psi_{i_*}(\cdot)$ satisfies the differential equation (2.6) for $\psi_i(T_i) = \psi_i^\circ$. The theorem is proved.

Problem 2. Find the times $T_1 = \ldots = T_N = T$, $T > t_0$, and admissible controls $u_{i_*}(\cdot) \in \mathrm{LU}_i$ such that the corresponding multivalued trajectories $X_i(\cdot, u_{i_*})$ of system (2.1)–(2.3) satisfy relationship (2.4) and the criterion (2.5) attains its minimum.

Denote by $R_0 = R^{K_1} \times \ldots \times R^{K_N}$ the Cartesian product of the Euclidean spaces $R^{K_1}, \ldots, R^{K_N}$. The elements x_0 of the space R_0 are written in the form $x_0 = (x_1, \ldots, x_N)$, $x_i \in R^{K_i}$.

We define the distance in the space R_0 by the formula

$$\rho(x_0, y_0) = \max\{d_i(x_i, y_i)\}, \quad x_0 = (x_1, \ldots, x_N), \quad y_0 = (y_1, \ldots, y_N)$$

Let us reduce the N-object Problem 2 to a problem with one fictitious object in the space R_0. We introduce the following notation in Problem 2 [1]

$$x_0 = (x_1, \ldots, x_N), \quad x_i \in R^n; \quad u_0 = (u_1, \ldots, u_N)$$
$$u_i \in R^{m_i}; \quad F_0(t, x_0, u_0) = F_1(t, x_1, u_1) \times \ldots \times F_N(t, x_N, u_N)$$

Then the system of differential inclusions (2.1)–(2.3) takes the form

$$x_0^{\cdot} \in F_0(t, x_0, u_0), \quad x_0(t_0) = x_0^{\circ}, \quad x_0^{\circ} = (x_1^{\circ}, \ldots, x_N^{\circ}) \tag{2.17}$$

Condition (2.4) may be rewritten in the form

$$\begin{gathered} H = X_0(T, u_0) \cap G \neq \varnothing \\ G = \{x_0 = (x_1, \ldots, x_N) \in R_0, \; x_1 = \ldots = x_N\} \\ X_0(T, u_0) = X_1(T, u_1) \times \ldots \times X_N(T, u_N) \end{gathered} \tag{2.18}$$

Clearly, if the mappings $F_i(\cdot, \cdot, \cdot)$ and $U_i(\cdot)$ satisfy conditions A1–A8, then the mappings $F_0(\cdot, \cdot, \cdot)$ and $U_0(\cdot) = U_1(\cdot) \times \ldots \times U_N(\cdot)$ also satisfy conditions A1–A8.

Like the above (see the reference cited in the footnote on p. 620), we can probe that Problem 2 is equivalent to the following problem.

Problem 3. Find the time $T > t_0$ and an admissible control $u_0(\cdot) \in \mathrm{LU}_0$ such that the multivalued trajectory $X_0(\cdot, u_0)$ of system (2.17) satisfies condition (2.18) and the criterion (2.5) attains its minimum.

Remark. If $u_0(\cdot) \in \mathrm{LU}_0$ is the optimal control in Problem 3, it can be represented in the form $u_0(\cdot) = (u_1(\cdot), \ldots, uN(\cdot))$, $u_i(\cdot) \in \mathrm{LU}_i$ and the controls $u_i(\cdot)$ are optimal in Problem 2.

If $u_i(\cdot) \in \mathrm{LU}_i$ are optimal in Problem 2, then the control $u_0(\cdot) = (u_1(\cdot), \ldots, u_N(\cdot))$ is optimal in Problem 3.

Definition 7. We say that the pairs $(u_{i_*}(\cdot), X_i(\cdot))$ satisfy the maximum principle in the interval $[t_0, T_i]$ if a vector $z \in R^n$ and vector functions $\psi_{i_*}(\cdot)$ exist which are respectively solutions of the system of differential inclusions (2.6) and the following conditions are satisfied:

1. The maximum condition, which differs from the maximum condition in Definition 6 in that it must hold for at least one i for almost all $t \in [t_0, T]$.
2. The transversality condition

$$C(X_i(T, u_{i*}), \psi_{i*}(T)) = (z, \psi_{i*}(T))$$

Theorem 3. Assume that conditions A1–A8 hold and that, in Problem 2, T is the minimum value of the functional (2.5), $u_{i_*}(\cdot) \in \mathrm{LU}_i$ is the optimal control and $X_i(\cdot, u_{i_*})$ are the corresponding multivalued trajectories of system (2.1)–(2.3). Then the pairs $(u_{i_*}(\cdot),\ X_i(\cdot, u_{i_*}))$ satisfy the maximum principle in $[t_0, T]$.

Proof. By the above remark, the control $u_{0_*} = (u_{1_*}(\cdot), \ldots, u_{N_*}(\cdot))$ is optimal and

$$X_0(\cdot, u_{0*}) = \prod_{i=1}^{N} X_i(\cdot, u_{i*})$$

is the corresponding multivalued trajectory in Problem 3. Therefore,

$$X_0(T, u_{0*}) \in Y_0(T),\ X_0(T, u_{0*}) \cup G \neq \varnothing$$

Following the same scheme as in the proof of Theorem 2, we obtain the existence of a vector function $\psi_{0_*}(\cdot)$—the solution of system (2.6)—such that

$$C(F_0(t, x_0(t), u_{0*}(t)),\ \psi_{0*}(t)) = \max_{u_0 \in U_0(t)} C(F_0(t, x_0(t), u_0), \psi_{0*}(t)) \tag{2.19}$$

for almost all $t \in [t_0, T]$

$$C(X_0(T, u_{0*}), \psi_{0*}(T)) = -C(G, -\psi_{0*}(T)) \tag{2.20}$$

Relationship (2.20) may be rewritten as

$$z = (\bar{x}_1, \ldots, \bar{x}_N),\ \bar{x}_1 = \ldots = \bar{x}_N = \bar{x},\ \bar{x} \in R^n \tag{2.21}$$

From (2.19) and (2.21) we obtain

$$C(X_i(T, u_{i*}), \psi_{i*}(T)) = (\bar{x}, \psi_{i*}(T))$$

and there exists at least one i such that condition (2.7) is satisfied for almost all $t \in [t_0, T]$; $\psi_{i_*}(\cdot)$ is the solution of system (2.6). The theorem is proved.

REFERENCES

1. PLOTNIKOVA L. I., One problem of optimal control. *Upravlyaemye Sistemy*. Nauka, Novosibirsk, **6**, 36–43, 1970.
2. ARTSTEIN Z., Weak convergence of set-valued functions and control. *SIAM J. Control* **13**, 4, 865–878, 1975.
3. BLAGODATSKIKH V. I., Maximum principle for differential inclusions. *Trudy Matem. Inst. Akad. Nauk SSSR* **166**, 23–43, 1984.

Translated by Z.L.